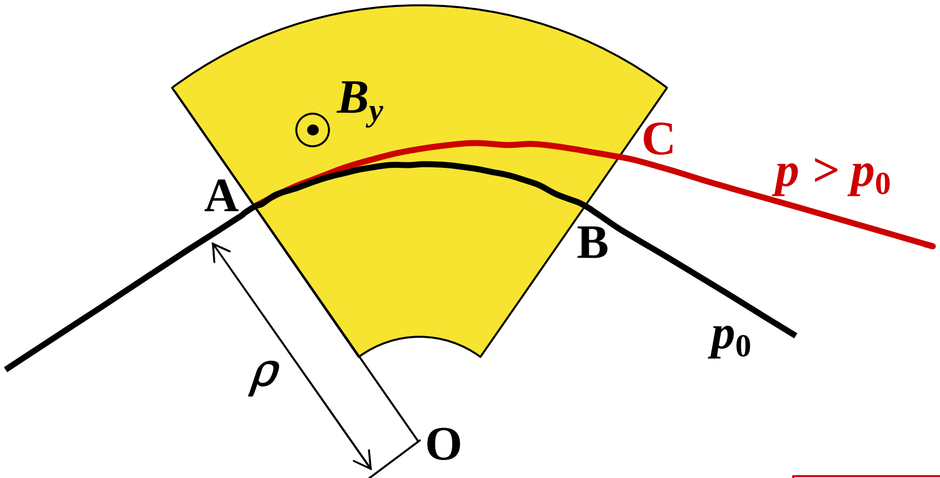
Lecture No. 8

Longitudinal Dynamics in Storage Rings

Fernando Sannibale

Path Length Dependence On Trajectory



$$\rho = \frac{p}{qB_z} = \frac{\beta\gamma m_0 c}{q B_z}$$

L_0 = Trajectory length between A and B

L = Trajectory length between A and C

$$\frac{L - L_0}{L_0} \propto \frac{p - p_0}{p_0} \quad \Rightarrow \quad \boxed{\frac{\Delta L}{L_0} = \alpha_C \frac{\Delta p}{p_0}}$$

where α_C is constant

$$\gamma = \frac{W}{m_0 c^2} = \frac{m_0 c^2 + E}{m_0 c^2} = 1 + \frac{E}{m_0 c^2}$$

$$\gamma \cong 1 + E_{[GeV]}/0.938 \quad \textit{for protons}$$

$$\gamma \cong 1 + E_{[MeV]}/0.511 \quad \textit{for electrons}$$

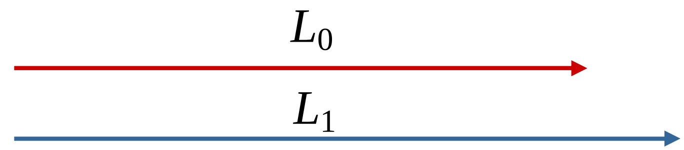
$$For \;\; \gamma >> 1 \;\; \Rightarrow \frac{\Delta L}{L_0} = \alpha_C \frac{\Delta p}{p_0} \cong \alpha_C \frac{\Delta E}{E_0}$$

In the example (sector bending magnet) $L > L_0$ so that $\alpha_C > 0$
Higher energy particles will leave the magnet later.

Path Length Dependence on Velocity

Consider two particles with different momentum on parallel trajectories:

$$p_1 = p_0 + \Delta p$$



At a given instant *t*:

$$L_1 = \left(\beta_0 + \Delta\beta\right) ct \qquad L_0 = \beta_0\, ct$$

$$\boxed{\Rightarrow \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0} = \frac{\Delta\beta}{\beta_0}}$$

But:

$$p = \beta\gamma m_0 c \quad \Rightarrow \quad \Delta p = m_0 c\, \Delta\left(\beta\gamma\right) = m_0 c \gamma^3 \Delta\beta$$

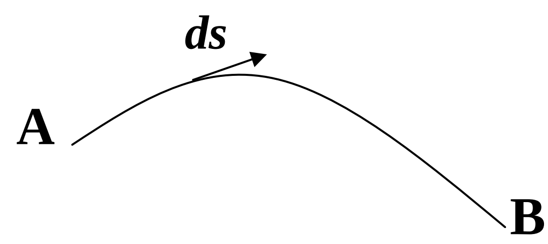
$$\Rightarrow \quad \frac{\Delta p}{p_0} = \gamma^2 \frac{\Delta\beta}{\beta}$$

$$\boxed{\frac{\Delta L}{L_0} = \frac{1}{\gamma^2} \frac{\Delta p}{p_0}}$$

- **This path length dependence on momentum applies everywhere, also in straight trajectories.**
- **The effect quickly vanishes for relativistic particles.**
- **Higher momentum particles precede the ones with lower momentum.**

Total Path Length Dependence on Momentum

• Let's consider a particle moving in a region in the presence of electric and magnetic fields. Under the action of such fields, the particle will define a trajectory of length L between the points A and B.



• We define as the **reference orbit** the trajectory of length L_0 that the **reference particle** with nominal energy E_0 describes between A and B. The position s of a generic particle will be referred to s_0, **the position of the reference particle on the reference orbit**:

$$\boxed{\Delta s = s - s_0} \qquad for\ \Delta s < 0\ the\ particle\ precedes\ the\ reference\ particle$$

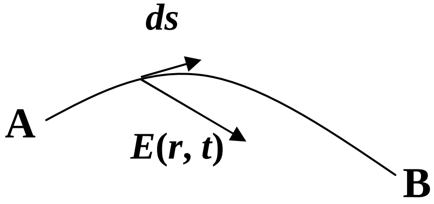
• In this reference frame we can combine the previous results and obtain for the **path length dependence on momentum**:

$$\boxed{\frac{\Delta s}{L_0} = -\left(\frac{1}{\gamma^2} - \alpha_C\right)\frac{\Delta p}{p_0} = -\eta_C \frac{\Delta p}{p_0}}$$

Where the constant $\eta_C = \gamma^2 - \alpha_C$ is called the **momentum compaction**

Energy Variation

• **The energy gain for a particle that moves from A to B is given by:**



$$\Delta E = q\int_0^L \bar{E}_F\left(\bar{r},t\right)\cdot d\bar{s} = qV$$

• **We define as V the voltage gain for the particle.**
V depends only on the particle trajectory and includes the contribution of every electric field present in the area (RF fields, space charge fields, fields due to the interaction with the vacuum chamber, …)

• **The particle can also experience energy variations $U(E)$ that depend also on its energy, as for the case of the radiation emitted by a particle under acceleration (synchrotron radiation when the acceleration is transverse).**

• **The total energy variation will be given by the sum of the two terms:**

$$\Delta E_T = qV + U\left(E\right)$$

The Rate of Change of Energy

The energy variation for the reference particle is given by:

$$\Delta E_T(s_0) = qV(s_0) + U(E_0)$$

For particle with energy $E = E_0 + \Delta E$ and orbit position $s = s_0 + \Delta s$:

$$\Delta E_T(s) = qV(s_0 + \Delta s) + U(E_0 + \Delta E) \cong qV(s_0) + q\left.\frac{dV}{ds}\right|_{s_0} \Delta s + U(E_0) + \left.\frac{dU}{dE}\right|_{E_0} \Delta E$$

Where the last expression holds for the case where $\Delta s \ll L_0$ (reference orbit length) and $\Delta E \ll E_0$.

In this approximation we can express the average rate of change of the energy respect to the reference particle energy by:

$$\frac{d\Delta E}{dt} \cong \frac{\Delta E_T(s) - \Delta E_T(s_0)}{T_0} \quad \Rightarrow \quad \boxed{\frac{d\Delta E}{dt} \cong \frac{1}{T_0}\left(q\left.\frac{dV}{ds}\right|_{s_0} \Delta s + \left.\frac{dU}{dE}\right|_{E_0} \Delta E\right)}$$

$$where \;\; T_0 = \frac{L_0}{\beta_0 c} \quad with \quad \begin{array}{l} L_0 = lenght\ of\ the\ reference\ orbit\ between\ A\ and\ B \\ \beta_0 c = velocity\ of\ the\ reference\ particle \end{array}$$

Towards the Longitudinal Motion Equation

In the present approximation of small Δs and ΔE, the average rate of change of the particle position respect to the reference particle position is:

$$\frac{d}{dt}\frac{\Delta s}{L_0} \cong -\frac{1}{T_0}\eta_C \frac{\Delta p}{p_0} \quad \Rightarrow \quad \boxed{\frac{d\Delta s}{dt} = -\beta_0 c \eta_C \frac{\Delta p}{p_0}} \quad \Rightarrow \quad \frac{d^2\Delta s}{dt^2} = -\frac{\beta_0 c \eta_C}{p_0}\frac{d\Delta p}{dt}$$

But:

$$dp = \frac{dE}{\beta c} \quad \Rightarrow \quad \Delta p \cong \frac{\Delta E}{\beta_0 c} \quad \Rightarrow \quad \frac{d^2\Delta s}{dt^2} = -\frac{\eta_C}{p_0}\frac{d\Delta E}{dt}$$

and remembering:

$$\boxed{\frac{d\Delta E}{dt} \cong \frac{1}{T_0}\left(q\left.\frac{dV}{ds}\right|_{s_0}\Delta s + \left.\frac{dU}{dE}\right|_{E_0}\Delta E \right)}$$

$$\frac{d^2\Delta s}{dt^2} = -\frac{\eta_C}{p_0}\frac{q}{T_0}\left.\frac{dV}{ds}\right|_{s_0}\Delta s - \frac{\eta_C}{p_0}\frac{1}{T_0}\left.\frac{dU}{dE}\right|_{E_0}\Delta E = -\frac{\eta_C}{p_0}\frac{q}{T_0}\left.\frac{dV}{ds}\right|_{s_0}\Delta s - \beta_0 c \eta_C \frac{\Delta p}{p_0}\frac{1}{T_0}\left.\frac{dU}{dE}\right|_{E_0}$$

$$\Rightarrow \quad \boxed{\frac{d^2\Delta s}{dt^2} = -\frac{\eta_C}{p_0}\frac{q}{T_0}\left.\frac{dV}{ds}\right|_{s_0}\Delta s + \frac{1}{T_0}\left.\frac{dU}{dE}\right|_{E_0}\frac{d\Delta s}{dt}}$$

The Longitudinal Motion Equation

Finally, by defining the quantities:

$$\Omega^2 = \eta_C \frac{1}{p_0} \frac{q}{T_0} \left.\frac{dV}{ds}\right|_{s_0}$$

$$\alpha_D = -\frac{1}{2T_0} \left.\frac{dU}{dE}\right|_{E_0}$$

We obtain the equations of motion for the longitudinal plane:

$$\frac{d^2\Delta s}{dt^2} + 2\alpha_D \frac{d\Delta s}{dt} + \Omega^2 \Delta s = 0$$

$$\Delta E(t) = -\frac{p_0}{\eta_C} \frac{d\Delta s}{dt}$$

$$\Delta s << L_0$$
$$\Delta E << E_0$$

We will study the case of storage rings where *dV/ds* is mainly due to the RF system used for restoring the energy lost per turn by the beam.

The Damped Oscillator Equation

$$\frac{d^2 \Delta s}{dt^2} + 2\alpha_D \frac{d\Delta s}{dt} + \Omega^2 \Delta s = 0$$

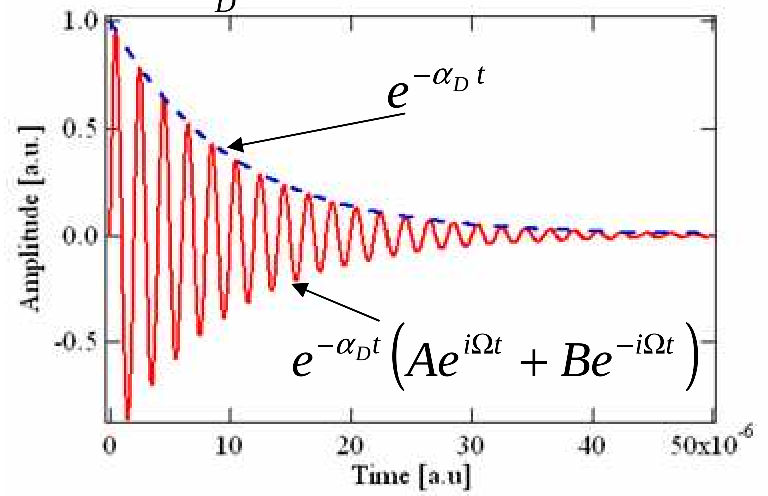
This expression is the well known damped harmonic oscillator equation, which has the general solution:

$$\Delta s(t) \cong e^{-\alpha_D t}\left(Ae^{i\Omega t} + Be^{-i\Omega t}\right)$$

$$\begin{array}{lll} \alpha_D > 0 & \Leftrightarrow & \text{damped oscillation} \\ \alpha_D < 0 & \Leftrightarrow & \text{anti-damped oscillation} \end{array}$$

$$\begin{array}{lll} \Omega^2 > 0 & \Leftrightarrow & \text{stable oscillation} \\ \Omega^2 < 0 & \Leftrightarrow & \text{unstable motion} \end{array}$$

$\alpha_D > 0 \quad and \quad \Omega^2 > 0$



The stable solution represents an oscillation with frequency $2\pi\Omega$ and with exponentially decreasing amplitude.

Damping in the Case of Storage Rings

- **The case of damped oscillations is exactly what we want for storing particles in a storage ring.**

$$\alpha_D > 0 \qquad \alpha_D = -\frac{1}{2T_0}\left.\frac{dU}{dE}\right|_{E_0} \quad \Rightarrow \quad \boxed{\left.\frac{dU}{dE}\right|_{E_0} < 0}$$

- **The synchrotron radiation (SR) emitted when particles are on a curved trajectory satisfies the condition. The SR power scales as:**

$$dU/dt = -P_{SR} \propto -(\beta\gamma)^4/\rho^2 = -\left(\gamma^2 - 1\right)^2/\rho^2 \qquad \rho \equiv trajectory\ \ radius$$

- **Typically, synchrotron radiation damping is very efficient in electron storage rings and negligible in proton machines.**

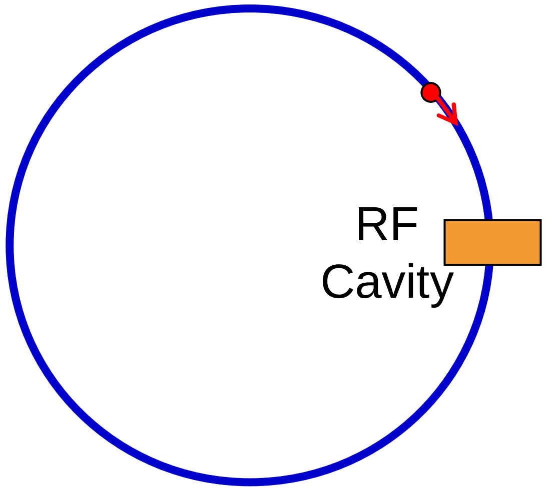
- **The damping time $1/\alpha_D$ (~ ms for e^-, ~ 13 hours LHC at 7 TeV) is usually much larger than the period of the longitudinal oscillations $1/2\pi\Omega$ (~ μs). This implies that the damping term can be neglected when calculating the particle motion for $t \ll 1/\alpha_D$:**

$$\boxed{\frac{d^2\Delta s}{dt^2} + \Omega^2 \Delta s = 0}$$

Harmonic oscillator equation

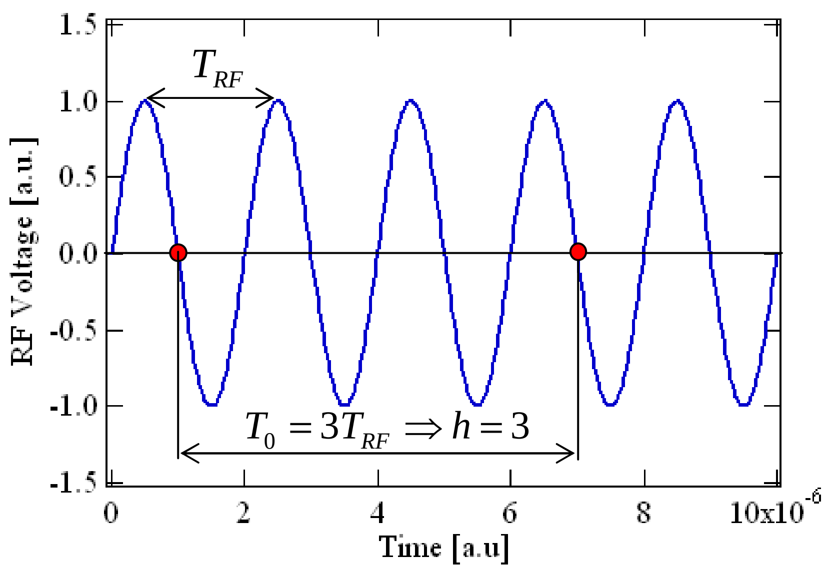
Synchronicity in Storage Rings

Let's consider a storage ring with reference trajectory of length L_0:



$$V_{RF}(t) = \hat{V}\sin(\omega_{RF}t)$$

$$T_0 = \frac{L_0}{\beta c} \qquad T_{RF} = \frac{1}{f_{RF}} = \frac{2\pi}{\omega_{RF}}$$



$$T_0 = hT_{RF} \Rightarrow f_0 = \frac{f_{RF}}{h}$$

Synchronicity Condition

The integer h is called the *harmonic number*

The Synchrotron Frequency and Tune

For our storage ring:

$$\Omega^2 = \eta_C \frac{1}{p_0} \frac{q}{T_0} \left.\frac{dV}{ds}\right|_{s_0} \qquad V_{RF}(t) = \hat{V}\sin(\omega_{RF}t) = \hat{V}\sin(h\,\omega_0\,t)$$

$$s = \beta_0\,c\,t \quad \Rightarrow \quad \left.\frac{dV}{ds}\right|_{s_0} = \frac{1}{\beta_0 c}\left.\frac{dV}{dt}\right|_{t_0} = \frac{h\omega_0\hat{V}}{\beta_0 c}\cos(\omega_{RF}t_0)$$

$$\boxed{\Omega^2 = \omega_0^2 \frac{q}{p_0} \frac{\eta_C h \hat{V}}{2\pi\beta_0 c}\cos(\varphi_s) \quad \textit{synchrotron frequency}} \qquad \boxed{\nu_S = \frac{\Omega}{\omega_0} \quad \textit{synchrotron tune}}$$

$$\boxed{\varphi_s = \omega_{RF}\,t_0 \equiv \textit{synchronous phase}}$$

In a storage ring the at equilibrium:

$$qV(s_0) + U(E_0) = q\hat{V}\sin(\varphi_s) - U_0 = 0 \quad \Rightarrow \quad \boxed{\sin\varphi_s = \frac{U_0}{q\hat{V}}}$$

Where U_0 is the energy lost per turn and V is integrated over turn.

The Synchrotron Oscillations

If $\alpha_D << \Omega$ ⇒ $\frac{d^2\Delta s}{dt^2} + \Omega^2 \Delta s = 0$ **Additionally:** $\Delta E(t) = -\frac{p_0}{\eta_C}\frac{d\Delta s}{dt}$

⇒ $\boxed{\Delta s = \Delta\hat{s}\cos(\Omega t + \psi)}$ $\boxed{\Delta E = \Delta\hat{s}\frac{p_0\Omega}{\eta_C}\sin(\Omega t + \psi)}$

A different set of variables:

$\boxed{Phase: \varphi = \phi - \phi_s}$ $\phi = \omega_{RF} t$ $s = \beta_0 c t$ ⇒ $s = \beta_0 c \frac{\phi}{\omega_{RF}}$

$\boxed{Relative\ Momentum\ Deviation: \ \delta = \frac{\Delta p}{p_0}}$ $\Delta E = \beta_0 c \Delta p$

$\boxed{\varphi = \hat{\varphi}\cos(\Omega t + \psi)}$ $\boxed{\delta = \frac{\hat{\varphi}\Omega}{h\omega_0\eta_C}\sin(\Omega t + \psi)}$

Synchrotron Oscillations

For $\Delta s << L_0$ and $\Delta E << E_0$.

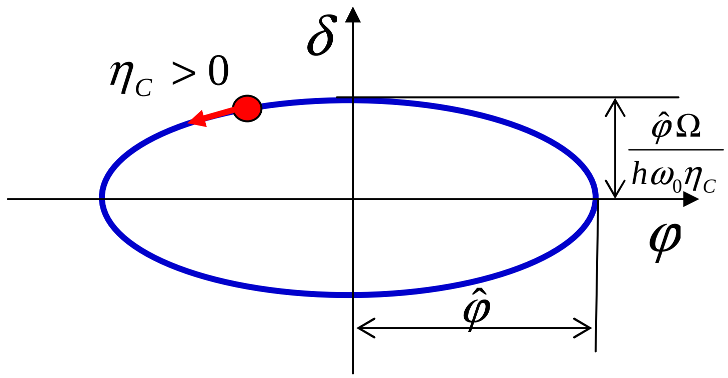
The Longitudinal Phase space

We just found:

$$\varphi = \hat{\varphi}\cos\left(\Omega t + \psi\right)$$

$$\delta = \frac{\hat{\varphi}\Omega}{h\omega_0\eta_C}\sin\left(\Omega t + \psi\right)$$

$$\frac{\varphi^2}{\hat{\varphi}^2} + \delta^2\left(\frac{h\omega_0\eta_C}{\hat{\varphi}\Omega}\right)^2 = 1$$

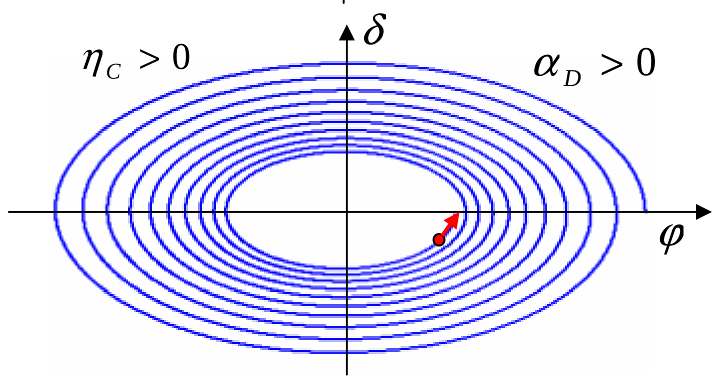


This equation represents an ellipse in the longitudinal phase space {φ, δ}

With damping:

$$\varphi = \hat{\varphi}e^{-\alpha_D t}\cos\left(\Omega t + \psi\right)$$

$$\delta = \frac{\hat{\varphi}\Omega}{h\omega_0\eta_C}e^{-\alpha_D t}\sin\left(\Omega t + \psi\right)$$



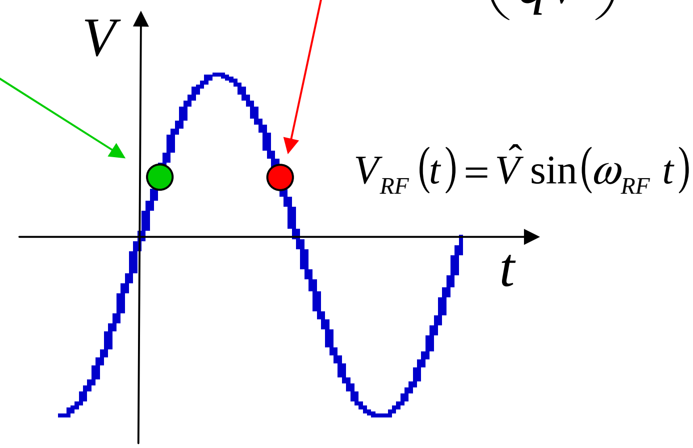
In rings with negligible synchrotron radiation (or with negligible non-Hamiltonian forces, the longitudinal emittance is conserved.

This is the case for heavy ion and for most proton machines.

Phase Stability

We showed that for the synchronous phase:

$$\sin\varphi_S = \frac{U_0}{q\hat{V}} \quad \Rightarrow \quad \varphi_S^1 = \arcsin\left(\frac{U_0}{q\hat{V}}\right) \quad or \quad \varphi_S^2 = \pi - \arcsin\left(\frac{U_0}{q\hat{V}}\right)$$



But

$$\frac{\Delta t}{T_0} = \frac{\Delta s}{L_0} = -\left(\frac{1}{\gamma^2} - \alpha_C\right)\frac{\Delta p}{p_0} = -\eta_C \frac{\Delta p}{p_0}$$

For positive charge particles:

$$For\ \eta_C > 0 \quad \Rightarrow \quad \varphi_S^1\ \ stable, \quad \varphi_S^2\ \ unstable$$

$$For\ \eta_C < 0 \quad \Rightarrow \quad \varphi_S^1\ \ unstable, \quad \varphi_S^2\ \ stable$$

For negative charge particles all the phases are shifted by π.

We define as *transition energy* the energy at which η_C changes sign.

$$\gamma_{TR} = \left(\frac{1}{\alpha_C}\right)^{\frac{1}{2}}$$

Crossing the transition energy during energy ramping requires a phase jump of ~ π

Large Amplitude Oscillations

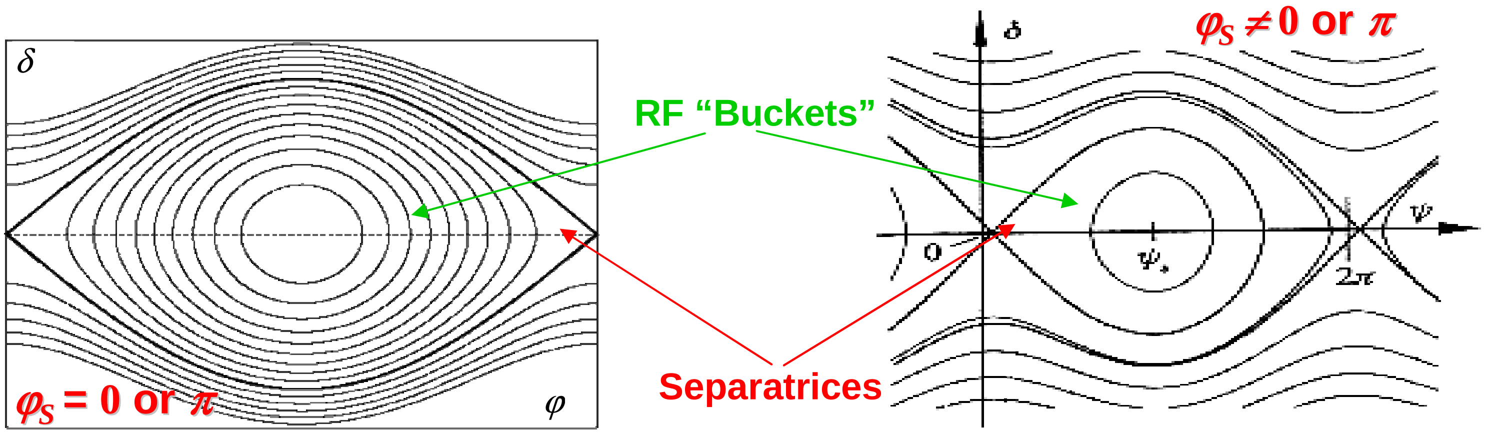
So far we have used the *small oscillation approximation* where:

$$\Delta E_T(\psi) = qV(\varphi_S + \varphi) = q\hat{V}\sin(\varphi_S + \varphi) \cong qV(\varphi_S) + q\left.\frac{dV}{d\varphi}\right|_{\varphi_S} \varphi = q\hat{V}\varphi_S + q\hat{V}\varphi$$

In the more general case of larger phase oscillations:

$$\Delta E_T(\psi) = qV(\varphi_S + \varphi) \cong q\hat{V}\sin(\varphi_S + \varphi)$$

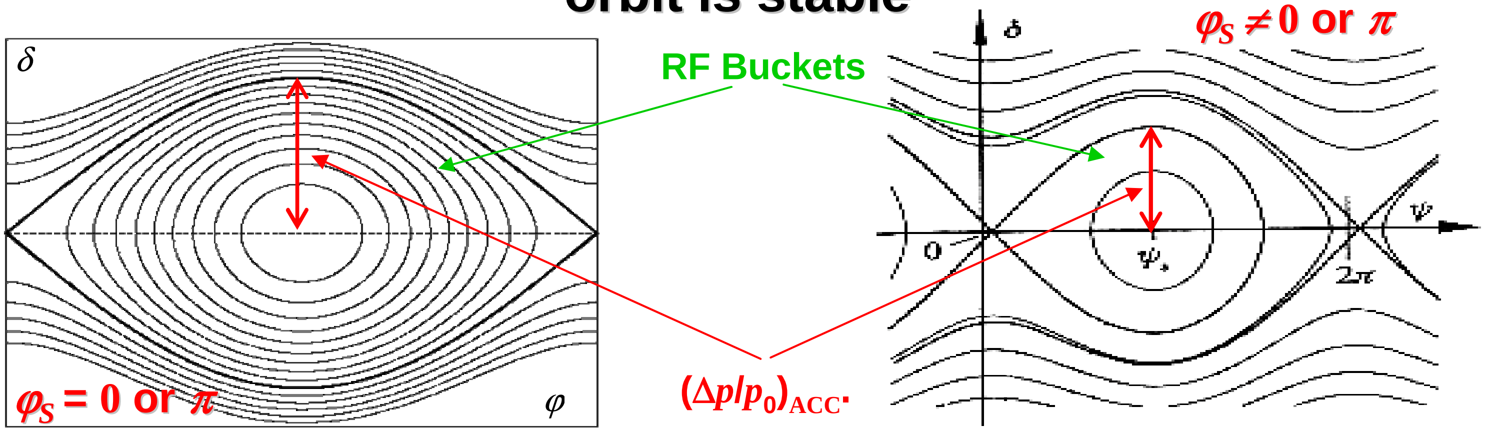
And by Numerical integration:



- **For larger amplitudes, trajectories in the phase space are not ellipsis anymore.**
- **Stable and unstable orbits exist. The two regions are separated by a special trajectory called *separatrix***
- **Larger amplitude orbits have smaller synchrotron frequencies**

Momentum Acceptance

The RF bucket is the area of the longitudinal phase space where a particle orbit is stable



The *momentum acceptance* is defined as the maximum momentum that a particle on a stable orbit can have.

$$\left(\frac{\Delta p}{p_0}\right)^2_{ACC} = \frac{2|q|\hat{V}}{\pi\, h |\eta_C| \beta\, c\, p_0}$$

$$\left(\frac{\Delta p}{p_0}\right)^2_{ACC} = \frac{F(Q)}{2Q}\frac{2|q|\hat{V}}{\pi\, h |\eta_C| \beta\, c\, p_0}$$

$$F(Q) = 2\left(\sqrt{Q^2 - 1} - \arccos\frac{1}{Q}\right)$$

$$Q = \frac{1}{\sin\varphi_s} = \frac{q\hat{V}}{U_0}$$

Over voltage factor

Bunch Length

• **In electron storage rings, the statistical emission of synchrotron radiation photons generates gaussian bunches.**

• **The over voltage Q is usually large so that the core of the bunch "lives" in the small oscillation region of the bucket. The equation of motion in the phase space are elliptical:**

$$\frac{\varphi^2}{\hat{\varphi}^2} + \delta^2 \left(\frac{h\omega_0 \eta_C}{\hat{\varphi}\Omega} \right)^2 = 1 \quad \Rightarrow \quad \hat{\varphi} = \frac{h\omega_0 \eta_C}{\Omega} \hat{\delta} \Rightarrow \Delta s = \frac{c\eta_C}{\Omega} \frac{\Delta p}{p_0}$$

• **If σ_p/p_0 is the *rms relative momentum spread* of the gaussian distribution, then the *rms bunch length* is given by:**

$$\boxed{\sigma_{\Delta S} = \frac{c\eta_C}{\Omega} \frac{\sigma_p}{p_0} = \sqrt{\frac{c^3}{2\pi q} \frac{p_0 \beta_0 \eta_C}{h\, f_0^2 \hat{V} \cos(\varphi_S)}} \frac{\sigma_p}{p_0}}$$

• **In the case of heavy ions and of most of protons machines, the whole RF bucket is usually filled with particles. The bunch length l is then proportional to the difference between the two extreme phases of the separatrix:**

$$\boxed{l = (\varphi_2 - \varphi_1) \lambda_{RF} / 2\pi}$$

Effects of the Synchrotron Radiation

- **A charged particle when accelerated radiates.**

- **In high energy storage rings transverse acceleration induces significant radiation (synchrotron radiation) while longitudinal acceleration generates negligible radiation ($1/\gamma^2$).**

$$\frac{dU}{dt} = -P_{SR} = -\frac{2\,c\,r_e}{3\left(m_0 c^2\right)^3}\frac{E^4}{\rho^2}$$

$r_e \equiv$ *classical electron radius*

$\rho \equiv$ *trajectory curvature*

$$U_0 = \int_{finite\ \rho} P_{SR}\, dt \quad \textit{energy lost per turn}$$

$$\alpha_D = -\frac{1}{2T_0}\frac{dU}{dE}\bigg|_{E_0} = \frac{1}{2T_0}\frac{d}{dE}\left[\oint P_{SR}(E_0)\,dt\right]$$

α_{DX}, α_{DY} *damping in all planes*

$\dfrac{\sigma_p}{p_0}$, $\varepsilon_X, \varepsilon_Y$ *equilibrium momentum spread and emittances*

- **Synchrotron radiation plays a major role in the dynamics of an electron storage ring**

Energy Lost per Turn

$$U_0 = \int_{finite\ \rho} P_{SR}\, dt \quad energy\ lost\ per\ turn \qquad \frac{dU}{dt} = -P_{SR} = -\frac{2\, c\, r_e}{3\left(m_0 c^2\right)^3} \frac{E^4}{\rho^2}$$

- **For relativistic electrons:**

$$s = \beta c t \cong ct \Rightarrow dt = \frac{ds}{c} \qquad \boxed{U_0 = \frac{1}{c} \int_{finite\ \rho} P_{SR}\, ds = \frac{2 r_e E_0^4}{3\left(m_0 c^2\right)^3} \int_{finite\ \rho} \frac{ds}{\rho^2}}$$

- **In the case of dipole magnets with constant radius ρ (*iso-magnetic* case):**

$$\boxed{U_0 = \frac{4\pi\, r_e}{3\left(m_0 c^2\right)^3} \frac{E_0^4}{\rho}}$$

- **The average radiated power is given by:**

$$\boxed{\left\langle P_{SR} \right\rangle = \frac{U_0}{T_0} = \frac{4\pi\, c\, r_e}{3\left(m_0 c^2\right)^3} \frac{E_0^4}{\rho\, L} \qquad L \equiv ring\ circumference}$$

Damping Coefficients

$$\frac{dU}{dt} = -P_{SR} = -\frac{2\,c\,r_e}{3\left(m_0c^2\right)^3}\frac{E^4}{\rho^2} \qquad \alpha_D = -\frac{1}{2T_0}\left.\frac{dU}{dE}\right|_{E_0} = \frac{1}{2T_0}\frac{d}{dE}\left[\oint P_{SR}(E_0)\,dt\right]$$

By performing the calculation one obtains:

$$\boxed{\alpha_D = \frac{U_0}{2T_0E_0}(2+D)}$$

Where *D* depends on the lattice parameters.
For the *iso-magnetic separate function* case:

$$D = \alpha_C \frac{L}{2\pi\rho}$$

Analogously, for the transverse plane:

$$\boxed{\alpha_X = \frac{U_0}{2T_0E_0}(1-D)} \quad \textbf{and} \quad \boxed{\alpha_Y = \frac{U_0}{2T_0E_0}}$$

Sometimes the *partition numbers* are used:

$$J_S = 2+D \qquad J_X = 1-D \qquad J_Y = 1 \qquad \textbf{with} \qquad \sum J_i = 4$$

Quantum Nature of Synchrotron Radiation

- **We saw that synchrotron radiation induces damping in all the planes.**
- **Because of that, one would expect that all the particles should collapse in a single point.**
- **This <u>does not</u> happen because of the quantum nature of synchrotron radiation.**
- **In fact, photons are randomly emitted in quanta of discrete energy and every time a photon is emitted the parent electron undergoes to a "jump" in energy.**
- **Such a process perturbs the electron trajectories exciting oscillations in all the planes.**
- **These oscillations grow until reaching <u>equilibrium</u> when balanced by the radiation damping.**

Emittance and Momentum Spread

- **At equilibrium the momentum spread is given by:**

$$\left(\frac{\sigma_p}{p_0}\right)^2 = \frac{C_q \gamma_0^2}{J_S} \frac{\oint 1/\rho^3 \, ds}{\oint 1/\rho^2 \, ds} \qquad where \; C_q = 3.84 \times 10^{-13} \, m$$

$$\left(\frac{\sigma_p}{p_0}\right)^2 = \frac{C_q \gamma_0^2}{J_S \rho}$$

$$iso-magnetic \;\; case$$

- **For the horizontal emittance at equilibrium:**

$$\varepsilon = C_q \frac{\gamma_0^2}{J_X} \frac{\oint H/\rho^3 ds}{\oint 1/\rho^2 ds}$$

where: $H(s) = \beta_T \eta'^2 + \gamma_T \eta^2 + 2\alpha_T \eta \eta'$

- **In the vertical plane, when no vertical bend is present, the synchrotron radiation contribution to the equilibrium emittance is very small and the vertical emittance is defined by machine imperfections and nonlinearities that couple the horizontal and vertical planes:**

$$\varepsilon_Y = \frac{\kappa}{\kappa+1}\varepsilon \quad and \quad \varepsilon_X = \frac{1}{\kappa+1}\varepsilon$$

$$with \quad \kappa \equiv \; coupling \;\; factor$$

Time Scale in Storage Rings

At this point we have discussed the motion of a particle in an accelerator for all the planes.

It can be helpful remarking the time scale for the different phenomena governing the particle dynamics.

Damping: several ms for electrons, ~ infinity for heavier particles

Synchrotron oscillations: ~ tens of μs

Revolution period: ~ hundreds of ns to μs

Betatron oscillations: ~ tens of ns

Possible Homework

- Calculate the general solution for the damped harmonic oscillator equation
- Calculate the ratio between the synchrotron radiation power radiated by a particle in the Large Hadron Collider (LHC), the proton collider at CERN, and the one radiated by a particle in the Advanced Light Source (ALS), the electron storage ring in Berkeley. The magnet bending radius is ~2810 m and ~5 m and the particle energy is 7000 GeV and 1.9 GeV for the LHC and the ALS respectively. (Remember that the electron mass is 9.1095 10^{-31} Kg while the proton one is 1.6726 10^{-27} Kg)
- Calculate the synchrotron frequency and tune for the ALS when the ring is operating in the following configuration: RF = 500 MHz, harmonic number = 328, E = 1.9 GeV, momentum compaction = 0.00137, energy lost per turn = 279 keV, peak RF voltage = 1.3 MV.
- Calculate the momentum acceptance for the ALS ring. Compare it with the acceptance value that the ring would have for zero synchronous phase.